COP 3503 - 3/12/2024

Exam 2 - Thursday

4 separate sheet

Calculator

3/13

* E2 Makeup 11:45am - 1:00pm Wednesday

3/12/2024

COP 3503 Spring 2022 Section 1 Exam #2

Sheet 1: Algorithm Analysis, Extra Sorts

Last Name: ____________________ **, First Name:** ______________________

Recitation Time (Circle One): **12:30** **1:30** **2:30** **3:30**

1) (5 pts) In a Bucket Sort of 100,000 real numbers, all in the range from [30, 130), if we decide to create 100,000 equally sized buckets, in which bucket number (in between 0 and 99,999 inclusive), would the number 40.2376 go? (Please show your work. The numbers have been chosen such that a calculator is not necessary.)

bucket size $= \frac{130-30}{10^5} = \frac{10^2}{10^5} = \frac{1}{1000} = .001$

$40.2376 - 30.0000 = 10.2376$

bucket 0 [0 to .001)

999 [.998, 1)

1000 [1 to 1.001]

$\left\lfloor \frac{10.2376}{.001} \right\rfloor$

10,237
____

2) (6 pts) Show the result of each iteration of a Radix Sort on the following values. The last column has been filled in.

Initial List	First Sort	Second Sort	Third Sort	Sorted List
8417	~~2341~~ 2431	3111	3111	2431
2431	3111	8417	8137	3111
8137	4461	2431 ~~8137~~	8417	4461
3111	8417	8137	2431	8137
4461	8137	4461	4461	8417

3) (14 pts) Consider the following game: You flip a coin. On the first flip it has a ½ chance of landing heads and a ½ chance of landing tails. If it lands heads, you stop. If it lands tails, you flip again. Unfortunately, the coin isn't kind. On the second flip, it will only land heads 1/3 of the time (and land tails 2/3 of the time). The game continues until you toss your first head. On the k[th] toss, the probability of getting heads is $\frac{1}{k+1}$. (Thus, the game is stacked against you, as it gets a little harder each time to flip a head!) Using the following steps, prove that the expected number of coin flips when playing this game is $\sum_{i=2}^{\infty} \frac{1}{i}$. (Note: this sum diverges, so the expected outcome is the game to go on forever!)

a) (1 pt) What is the probability you stop playing after 1 turn? $\frac{1}{2}$

b) (2 pts) What is the probability you stop playing after exactly 2 turns? $\frac{1}{6}$

$\frac{1}{2} \times \frac{1}{3}$ (T H)

$\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$

c) (2 pts) What is the probability you stop playing after exactly 3 turns? $\frac{1}{12}$

d) (4 pts) Generalize the result for k turns. What is the probability you stop playing after exactly k turns, where k is a positive integer? (Your work should include a product of fractions. You may informally express this product and simplify it to relatively simple looking fraction.)

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \frac{k-1}{k} \times \frac{1}{k+1} = \frac{1}{k(k+1)}$

T T T T H

k-1 tails

$\frac{1}{k(k+1)}$

e) (5 pts) Now, apply the formula derived in part (d) and plug it into the expectation formula to create an expression that is equal to the expected number of coin flips when playing this game. Show your work and simplify your expression to the summation shown in the question.

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} \times k = \sum_{k=1}^{\infty} \frac{1}{k+1} = \sum_{i=2}^{\infty} \frac{1}{i} \checkmark$$

p(ending k turns)

end start
13 > 1,2

1,2 → 9,10,10 | 11 11 13 13 | 13 | 13

Sheet 2: Greedy Algorithms

Last Name: ____________________ **, First Name:** ______________________

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4) (15 pts) Consider the following problem: given a list of ***n*** integers, determine if it's possible to place ***k*** of those integers in a set **S** and ***m*** of the integers (all different from the ones chosen to be in **S**) in a set **T** such that $1 \le k < m$ but the sum of the integers in **S** is strictly greater than the sum of the integers in **T**. Write a static method that solves this problem taking in an input array of integers. You may assume that the sum of the integers in the array doesn't exceed 10^9. To get full credit, your method must run in O(***n*lg*n***) time. The function prototype is below. You may assume the input array is at least size 3 (so you don't have to worry about an array out of bounds in the initial code.) **Note: The input array can be in any initial order.**

(positive)

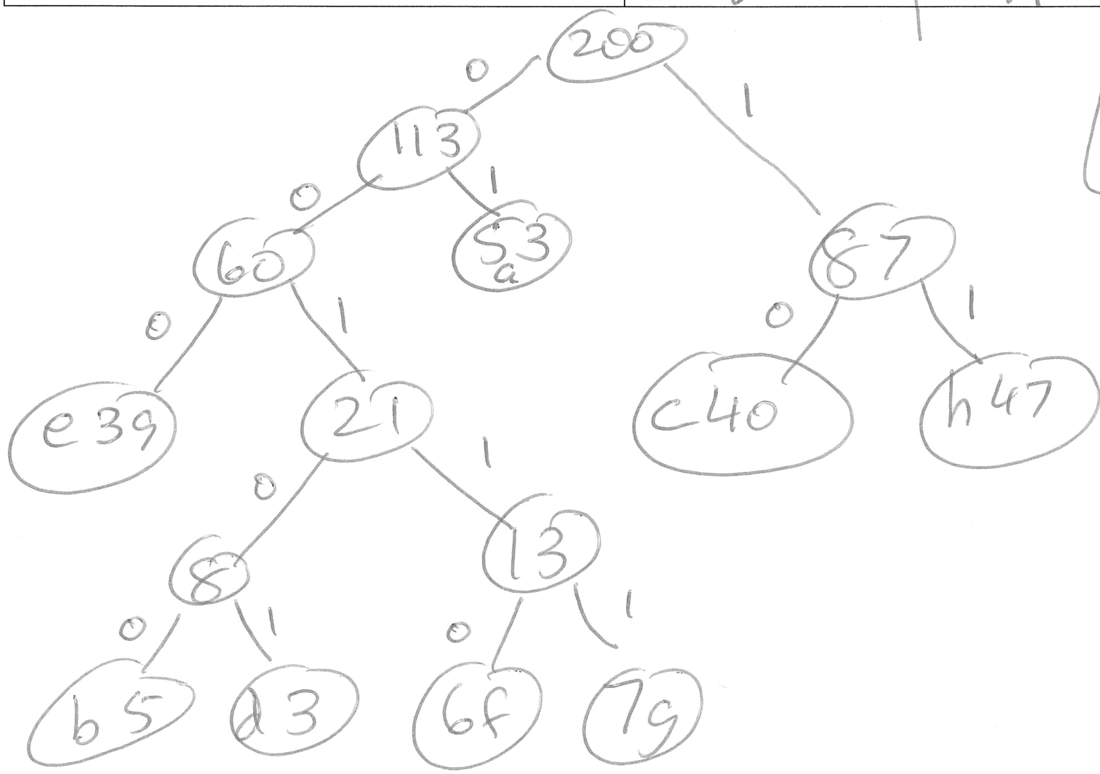
Counter-ex
1, 2, 13, 13 | 13, 13

```
public static boolean canSplit(int[] a) {

    Arrays.sort(a);
    int s = 0, t = a[0];
    for (int i=a.length-1, j=1; j<i; i--, j++) {
        s += a[i];
        t += a[j];
        if (s > t) return true;
    }
    return false;

    return s > t;   (is also ok)

}
```

5) (10 pts) Draw a valid Huffman tree for a file that contains the following letter frequencies. Draw a circle around your final answer. In the interest of time, no need to give the codes for each letter or anything else. Just draw the tree. In each leaf node, write the letter and its frequency. In each internal node, just write the sum of frequencies in the corresponding subtree.

Letter	Frequency	CODE
A	53	01
B	5	00100 X
C	40	10
D	3	00101
E	39	000
F	6	00110
G	7	00111
H	47	11

ans = 3 bits each



bits saved = 53 − 2(5) + 40 − 2(3) − 2(6) − 2(7) + 47

= 140 − ~~40~~ 42 = ~~100~~

98

Sheet 3: Unweighted Graph Algorithms

Last Name: ____________________, First Name: ______________________

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6) (8 pts) In a breadth first search of 8 Puzzle positions, what are the four reachable board positions (in one move) from the following board state? Draw out each answer.

6	3	5
2		7
1	8	4

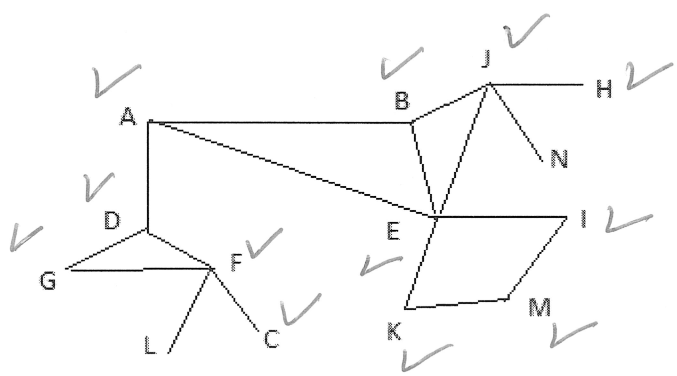
6		5
2	3	7
1	8	4

6	3	5
	2	7
1	8	4

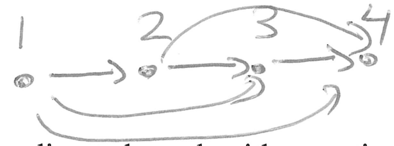
6	3	5
2	7	
1	8	4

6	3	5
2	8	7
1		4

7) (7 pts) Show the order in which the vertices in the following graph get visited in a DFS starting at vertex A. Whenever looping through the neighbors of a vertex, always go through those in alphabetical order.



A, B, E, I, M, K, J, H, N, D, F, C, G, L



8) (10 pts) Let G be a directed graph with ***n*** vertices which has precisely one possible topological sort.

(a) (1 pt) What is the fewest number of edges that could be in G, in terms of ***n***? n-1

(b) (2 pts) Give an informal proof that the answer in (a) is minimal.

With n-2 edges only n-2 vertices can have an incoming edge, leaving 2 vertices that could be the 1st item in a top sort, so if one exists it won't be unique.

(c) (3 pts) What is the maximum number of edges that could be in G, in terms of ***n***?

Can only draw 1 edge btw any pair of vertices. # pairs = $\binom{n}{2} = \boxed{\frac{n(n-1)}{2}}$

____

(d) (4 pts) Give an informal proof that answer in (c) is maximal.

If we had more than $\binom{n}{2}$ edges then at least 1 pair of vertices would have 2 edge btw them. But if this is the case then $i \to j$ and $j \to i$ and no top sort can exist.

Sheet 4: Weighted Graph Algorithms

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9) (10 pts) Show the contents of the distance array after each iteration of Dijkstra's algorithm for the directed graph G, with starting vertex A, whose edges are designated in the adjacency matrix below. (The row represents starting vertex of an edge and the column the ending vertex of that edge.) In addition, for each iteration, show the new vertex that gets pulled from the priority queue, indicating that the current distance stored to it is shortest. If an entry is missing, there is no edge between those vertices. **(Note: to get full credit you must fill in every square that is empty in the answer grid.)**

From/To	A	B	C	D	E	F
A		***27***	13	99	15	3
B			2	8	6	3
C		3		30	1	1
D		8	7		20	
E		9	30	2		10
F		20	12	27	9	

Note: The 27 that is in bold and italics represents that there is an edge from A to B with weight 27. There is no edge from B to A. This note is to clarify the direction of the edges described in the table.

Place Your Answer Below

Add	A	B	C	D	E	F
None	0	∞	∞	∞	∞	∞
A	0	27	13	99	15	3
F	0	23	13	30	12	3
E	0	21	13	14	12	3
C	0	16	13	14	12	3
D	0	16	13	14	12	3

10) (10 pts) A university has a set of professors, $\{P_1, P_2, P_3, \ldots, P_n\}$ and a set of classes $\{C_1, C_2, C_3, \ldots, C_m\}$. Professor P_i must teach exactly a_i classes from a set S_i of classes they know how to teach. Note that the sum of the a_i's equals m (so the # of classes the professors are required to cover equals exactly the number of courses.) Describe how to set up a network flow graph that determines whether it's possible to assign the professors to cover all of the courses. Draw a picture of a portion of the graph for one professor, P_1 who must teach 3 courses from the set C_1, C_3, C_6, C_{10} and C_{12}. Also, explain how, if we know the max flow of the graph, we can determine whether or not all of the classes can be covered.

Create a network for network flow. In addition to source + sink, each professor and each class get a vertex. Add an edge from source to each professor w/ a capacity equal to the number of classes they are assigned to teach. Add an edge from each professor to each class they can teach with capacity 1. Add an edge from each class to the sink with capacity 1. If the max flow of this graph equals m, then all classes can be covered, otherwise they can't.

11) (5 pts) NASA's Space Launch System rocket with Orion spacecraft was rolled out to launchpad 39B at Kennedy Space Center last week for some tests. What acronym is used to describe the Space Launch System?

SLS